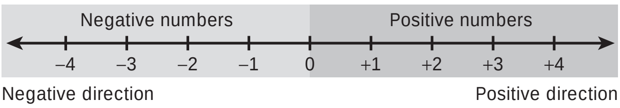
Directed Numbers

(S1A Chapter 1)

- Positive numbers
  Numbers greater than zero, e.g. $+1,\ +4,\ +2.9,\ +\frac{2}{7}$.
- Negative numbers
  Numbers smaller than zero, e.g. $-2,\ -8,\ -11.3,\ -1\frac{9}{5}$.
- A directed number is a number with a positive or negative sign before it.
- Number line



01

Directed Numbers

(S1A Chapter 1)

- Rules for removing brackets
  $A + (+B) = A + B$
  $A - (+B) = A - B$
  $A + (-B) = A - B$
  $A - (-B) = A + B$
- Rules for multiplication of directed numbers
  $(+)(+) = (+)$ $\quad (+)(-) = (-)$ $\quad (-)(+) = (-)$ $\quad (-)(-) = (+)$
- Rules for division of directed numbers
  $\frac{(+)}{(+)} = (+)$ $\quad \frac{(+)}{(-)} = (-)$ $\quad \frac{(-)}{(+)} = (-)$ $\quad \frac{(-)}{(-)} = (+)$

02

Basic Algebra

(S1A Chapter 2)

- Algebraic expressions are expressions obtained by addition, subtraction, multiplication or division of numbers and letters representing numbers.
- Formula is an equality representing the relations between numbers.
- The method of replacing letters with numbers in algebraic expressions is called substitution.

03

Basic Algebra

(S1A Chapter 2)

- A set of numbers arranged in a particular order forms a sequence. Each number in the sequence is called a term of the sequence.
- Square sequence
  1, 4, 9, 16, ⋯
- Triangular sequence
  1, 3, 6, 10, ⋯
- Arithmetic sequence
  An arithmetic sequence is a sequence in which the difference of any term (except the first term) minus its preceding one is the same.
- Geometric sequence
  A geometric sequence is a sequence in which the quotient of dividing any term (except the first term) by its preceding one is the same.
- Fibonacci sequence
  1, 1, 2, 3, 5, 8, 13, ⋯

04

Basic Geometry

(S1A Chapter 3)

- Triangles
  (a) Scalene triangle, isosceles triangle and equilateral triangle can be classified according to the lengths of their sides.
  (b) Acute-angled triangle, right-angled triangle and obtuse-angled triangle can be classified according to the sizes of their interior angles.
- Polygons
  (a) Convex polygon – a polygon with all interior angles smaller than 180°.
  (b) Concave polygon – a polygon with at least one interior angle greater than 180°.
  (c) Equilateral polygon – a polygon with all sides of equal length.
  (d) Equiangular polygon – a polygon with all equal interior angles.
  (e) Regular polygon – a polygon with all equal angles and sides.

05

Basic Geometry

(S1A Chapter 3)

- Uniform cross-section
  If we cut a solid along one direction, the cross-sections obtained at any positions are the same in both shape and size, these cross-sections are called uniform cross-sections.
- Polyhedra
  Polyhedra are closed solids which are formed by polygons.
- Euler's formula
  In a polyhedron, if the number of vertices, edges and faces is V, E and F respectively, then $V - E + F = 2$.

06

Example

Given a sequence 4, 8, 12, 16, ⋯,

(a) find an algebraic expression to represent the nth term of the sequence.

(b) Using the result of **(a)**, find the 10th term of the sequence.

Solution:

(a) The 1st term $= 4 = 4 \times 1$

The 2nd term $= 8 = 4 \times 2$

The 3rd term $= 12 = 4 \times 3$

The 4th term $= 16 = 4 \times 4$

According to the pattern,

the nth term $= 4 \times n$

$= \underline{\underline{4n}}$

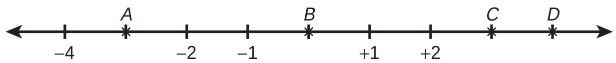
(b) The 10th term of the sequence $= 4 \times 10$

$= \underline{\underline{40}}$

Example

(a) If '+1 m' represents 1m to the south, express 3 m to the north in positive or negative numbers.

(b) According to the number line below, write down the values of points A to D.

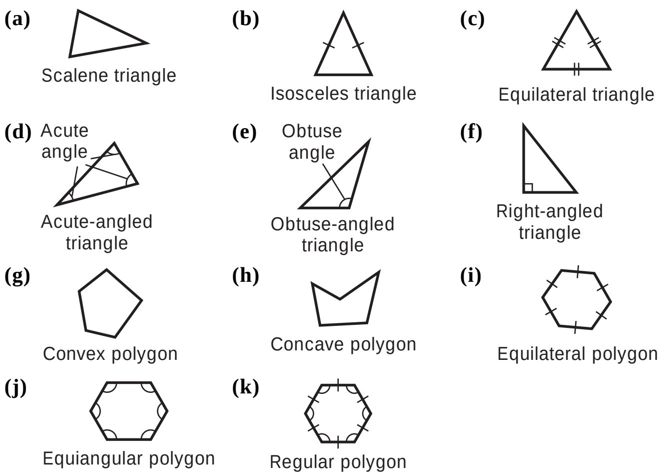


Solution:

(a) '3 m to the north' is expressed as $\underline{\underline{-3\text{ m}}}$.

(b) $A = -3$, $B = 0$, $C = +3$, $D = +4$

Example



Example

Evaluate the following.

(a) $(+2) + (-3)$

(b) $(-4) - (-2)$

(c) $(-10)\ (+2)$

(d) $(-8) \div (+4)$

Solution:

(a) $(+2) + (-3) = 2 - 3$

$= \underline{\underline{-1}}$

(b) $(-4) - (-2) = -4 + 2$

$= \underline{\underline{-2}}$

(c) $(-10)(+2) = -(10 \times 2)$

$= \underline{\underline{-20}}$

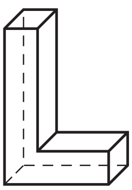
(d) $(-8) \div (+4) = -\dfrac{8}{4}$

$= \underline{\underline{-2}}$

Example

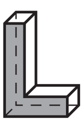
(a) Colour the uniform cross-section of the solid.

(b) Does the solid satisfy the Euler's formula?



Solution:

(a)



(b) $V = 12$, $E = 18$, $F = 8$

$\because$ $V - E + F = 12 - 18 + 8 = 2$

$\therefore$ The solid satisfies the Euler's formula.

Example

If $x = 8$ and $y = 5$, find the values of the following expressions.

(a) $x + 2y$

(b) $(x - y)^2$

Solution:

(a) $x + 2y = 8 + 2(5)$

$= 8 + 10$

$= \underline{\underline{18}}$

(b) $(x - y)^2 = (8 - 5)^2$

$= 3^2$

$= \underline{\underline{9}}$

Linear Equations in One Unknown

(S1A Chapter 4)

- If an equation has one unknown only and the unknown is of degree 1, the equation is called a linear equation in one unknown.
- If two sides of an equation are equal after substituting a value for the unknown of the equation, the value is called the solution (or root) of the equation.
- When solving an equation, the unknown can be found by applying the same operation to both sides of the equation.

07

Percentages

(S1A Chapter 5)

- Percentages are fractions with 100 as the denominator.
- Percentage increase

  (a) Percentage increase $= \dfrac{\text{Increase}}{\text{Original value}} \times 100\%$

  (b) Increase = Original value × Percentage increase

  (c) New value = Original value + Increase
  = Original value × (1 + Percentage increase)
- Percentage decrease

  (a) Percentage decrease $= \dfrac{\text{Decrease}}{\text{Original value}} \times 100\%$

  (b) Decrease = Original value × Percentage decrease

  (c) New value = Original value − Decrease
  = Original value × (1 − Percentage decrease)

08

Percentages

(S1A Chapter 5)

- Profit

  (a) Profit = Selling price − Cost

  (b) Percentage profit $= \dfrac{\text{Profit}}{\text{Cost}} \times 100\%$

  (c) Selling price = Cost × (1 + Percentage profit)
- Loss

  (a) Loss = Cost − Selling price

  (b) Percentage loss $= \dfrac{\text{Loss}}{\text{Cost}} \times 100\%$

  (c) Selling price = Cost × (1 − Percentage loss)
- Discount

  (a) Discount = Marked price − Selling price

  (b) Percentage discount $= \dfrac{\text{Discount}}{\text{Marked price}} \times 100\%$

  (c) Selling price = Marked price × (1 − Percentage discount)

09

Statistics in Daily Life

(S1A Chapter 6)

- Four stages involved in statistics:

  Collection of data → Organization of data → Presentation of data → Analysis of data

- The following are five common data collection methods:

  (a) Reading records

  (b) Observation

  (c) Experiment

  (d) Interview

  (e) Questionnaire

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Statistics in Daily Life

(S1A Chapter 6)

- Diagrams to present data

  (a) Pictograms

  (b) Bar charts

  (c) Compound bar charts

  (d) Broken-line graphs

  (e) Pie charts

  (f) Stem-and-leaf diagrams

11

Algebraic Expressions and Polynomials

(S1B Chapter 7)

- Monomial

  (a) An algebraic expression has only one term and the term is

  (i) a number, or

  (ii) the product of a number and variable(s) with indices, where the indices are positive integers.

  (b) In a monomial, the coefficient of a term is the numerical part attached to the variable(s). A term without any variables is called a constant term.

  (c) The sum of the indices of the variables is called the degree of the monomial.
- Polynomials

  (a) A monomial or the sum of monomials is called a polynomial.

  (b) In a polynomial, the highest degree of all terms is the degree of the polynomial.

12

Example

A survey is conducted by a travel agency to investigate the most favourite tourist spot with local people. The steps in order are as follows:

I. Interviewing the local people and ask them to vote for their favourite tourist spots.

II. Counting the number of votes for each tourist spot.

III. Using a pie chart to show the percentage of votes for each favourite tourist spot.

IV. Listing the top three favourite tourist spots with local people.

Example

Jane bought eight story books for $\$x$ each. She paid \$700 and got a change of \$272.

(a) Set up an equation to find x.

(b) Find the selling price of each story book.

Solution:

(a) The required equation is $\underline{\underline{700-8x=272}}$.

(b)
$$\begin{aligned}700-8x&=272\\700-272&=8x\\8x&=428\\x&=53.5\end{aligned}$$

$\therefore$ The selling price of each story book is \$53.5.

Example

The following stem-and-leaf diagram shows the results of 20 candidates in an examination.

Results of 20 candidates

Stem (10 marks)	Leaf (1 mark)
5	3 4 6 7
6	2 5 5 6 8
7	0 2 4 5 7 7 9
8	1 3 5 6

(a) Write down the highest mark of 20 candidates in the examination.

(b) How many candidates have obtained marks below 60?

Solution:

(a) The highest mark of 20 candidates is 86 marks in the examination.

(b) 4 candidates have obtained marks below 60.

Example

Find the new value after the change in each of the following.

(a) Increase 160 m by 20%

(b) Decrease 280 L by 1%

Solution:

(a) Increase = $160 \times 20\%$ m = 32 m

$\therefore$ New value $=(160+32)\text{ m}=\underline{\underline{192\text{ m}}}$

(b) Decrease = $280 \times 1\%$ L = 2.8 L

$\therefore$ New value $=(280-2.8)\text{ L}=\underline{\underline{277.2\text{ L}}}$

Example

(a) Simplify $(x-5)-(2x+4)+(2+3x^2)$.

(b) Expand $(3x^2-5)(2x-1)$.

Solution:

(a)
$$\begin{aligned}&(x-5)-(2x+4)+(2+3x^2)\\&=x-5-2x-4+2+3x^2\\&=3x^2+x-2x-5-4+2\\&=\underline{\underline{3x^2-x-7}}\end{aligned}$$

(b)
$$\begin{aligned}&(3x^2-5)(2x-1)\\&=(3x^2)(2x-1)+(-5)(2x-1)\\&=\underline{\underline{6x^3-3x^2-10x+5}}\end{aligned}$$

Example

The marked price of a mobile phone is \$2 500. A shopkeeper sells it for \$2 250.

(a) Find the percentage discount of the mobile phone.

(b) It is known that the percentage profit of the mobile phone sold for \$2 250 is 25%. Find the cost of the mobile phone.

Solution:

(a) Discount = \$2 500 − \$2 250 = \$250

Percentage discount $=\dfrac{250}{2\ 500}\times 100\%=\underline{\underline{10\%}}$

(b) Let the cost of the mobile phone be $\$C$.

$$\begin{aligned}C(1+25\%)&=2\ 250\\C&=2\ 250\div 1.25\\&=1\ 800\end{aligned}$$

$\therefore$ The cost of the mobile phone is \$1 800.

Algebraic Expressions and Polynomials

(S1B Chapter 7)

- Concept of powers

  If n is a positive integer, then $a^n = \underbrace{a \times a \times a \times \cdots \times a}_{n \text{ times}}$

- Operations of powers

  For positive integers m and n,

  (a) $a^m \times a^n = a^{m+n}$

  (b) $a^m \div a^n = a^{m-n}$ (where $a \neq 0$ and $m > n$)

  (c) $a^m \div a^n = \dfrac{1}{a^{n-m}}$ (where $a \neq 0$ and $m < n$)

  (d) $(a^m)^n = a^{mn}$

  (e) $(ab)^n = a^n b^n$

13

Symmetry and Transformation

(S1B Chapter 8)

- Reflectional symmetry

  If a plane figure is folded along a line so that the two sides coincide, the figure has the property of reflectional symmetry, and the line is called the axis of symmetry.

- Rotational symmetry

  If a plane figure rotates about a point for 1 turn and coincides with the original figure 2 times or more, this plane figure has the property of rotational symmetry, and the point is called the centre of rotation.

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Symmetry and Transformation

(S1B Chapter 8)

- Transformation of plane figure

Transformation	Property
Translation	Change the position of a figure, but its shape, size and orientation remain unchanged.
Reflection	Change the orientation of a figure, but its shape and size remain unchanged.
Rotation	Change the orientation of a figure, but its shape and size remain unchanged.
Enlargement / Contraction	Change the size of a figure, but its shape and orientation remain unchanged.

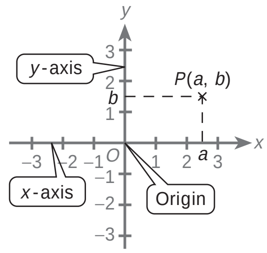
15

Introduction to Coordinates

(S1B Chapter 9)

- Rectangular coordinate

  The figure is a rectangular coordinate plane. It is formed by a horizontal line (x-axis) and a vertical line (y-axis).

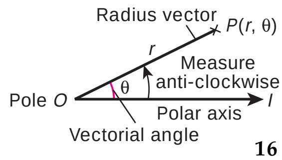


  (a) The x-axis and y-axis are called the coordinate axes, their point of intersection is called the origin.

  (b) The position of any point P on the plane can be represented by an ordered pair (a, b). a and b are the x-coordinate and y-coordinate of the point respectively.

- Polar coordinate

  The polar coordinate system is a coordinate system which uses the distance and angle of a reference point to determine the positions of points.



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Introduction to Coordinates

(S1B Chapter 9)

- Transformations on a rectangular coordinate plane

Transformation	Coordinates of the image
(x, y) is translated h units to the right and k units upwards	$(x + h, y + k)$
(x, y) is reflected along the x-axis	$(x, -y)$
(x, y) is reflected along the y-axis	$(-x, y)$
(x, y) is rotated anti-clockwise about the origin through 90°	$(-y, x)$
(x, y) is rotated clockwise about the origin through 90°	$(y, -x)$
(x, y) is rotated about the origin through 180°	$(-x, -y)$

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Statistical Graphs

(S1B Chapter 10)

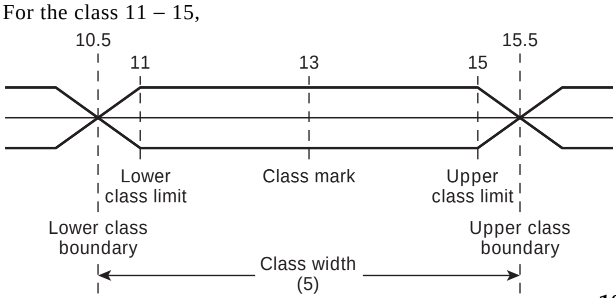
- Types of data

  (a) Discrete data

  (b) Continuous data

- Concepts about classes

  For the class 11 – 15,



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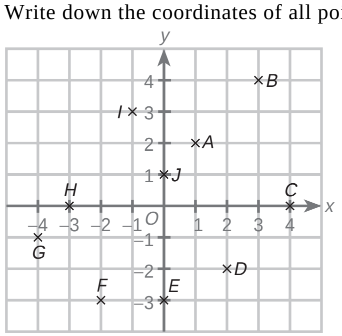
Revision Cards S1

Example

Write down the coordinates of all points in the figure.



Solution:

$\underline{\underline{A(1,\ 2),\ B(3,\ 4),\ C(4,\ 0),\ D(2,\ -2),\ E(0,\ -3),\ F(-2,\ -3),\ G(-4,\ -1),}}$

$\underline{\underline{H(-3,\ 0),\ I(-1,\ 3),\ J(0,\ 1)}}$

Example

Simplify the following.

(a) $ab^2 \times a^3$ **(b)** $\dfrac{(a^2b)^2}{ab^3}$

Solution:

(a) $ab^2 \times a^3 = a^{1+3}b^2 = \underline{\underline{a^4b^2}}$

(b) $\dfrac{(a^2b)^2}{ab^3} = \dfrac{(a^2)^2b^2}{ab^3} = \dfrac{a^{2\times2}b^2}{ab^3} = \dfrac{a^4b^2}{ab^3} = \dfrac{a^{4-1}}{b^{3-2}} = \underline{\underline{\dfrac{a^3}{b}}}$

Example

Given $A(3,6)$, find the coordinates of the images of A after the following transformations.

(a) Translate 4 units to the right and 2 units downwards.

(b) Reflect along the x-axis.

(c) Reflect along the y-axis.

(d) Rotate clockwise about the origin through 90°.

Solution:

(a) Coordinates of the image of $A = (3+4,\ 6-2) = \underline{\underline{(7,\ 4)}}$

(b) Coordinates of the image of $A = \underline{\underline{(3,\ -6)}}$

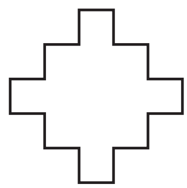
(c) Coordinates of the image of $A = \underline{\underline{(-3,\ 6)}}$

(d) Coordinates of the image of $A = \underline{\underline{(6,\ -3)}}$

Example

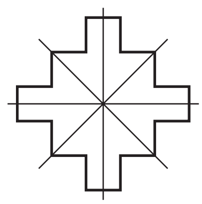
(a) Draw all axes of symmetry and write down the number of axes of symmetry.

(b) Mark the centre of rotation and write down the number of folds of rotational symmetry.



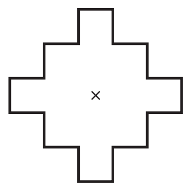
Solution:

(a)



$\underline{\underline{\text{4 axes of symmetry}}}$

(b)



$\underline{\underline{\text{4-fold rotational symmetry}}}$

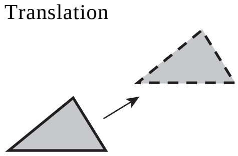
Example

(a) Discrete data
e.g. number of product, number of people ⋯ etc.

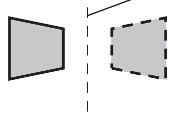
(b) Continuous data
e.g. length, weight, time ⋯ etc.

Example

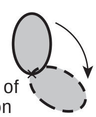
(a) Translation



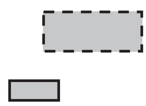
(b) Reflection



(c) Rotation



(d) Enlargement



Statistical Graphs
(S1B Chapter 10)

- Diagrams for presenting data
  (a) Histograms
  (b) Scatter Diagrams

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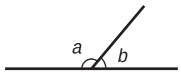
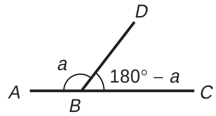
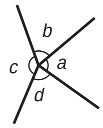
Angles in Rectilinear Figures
(S1B Chapter 13)



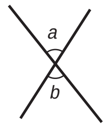
$a + b = 180°$
[adj. ∠s on st. line]



ABC is a straight line
[adj. ∠s supp.]



$a + b + c + d = 360°$
[∠s at a pt.]



$a = b$
[vert. opp. ∠s]

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Linear Equations in Two Unknowns
(S1B Chapter 11)

- If an equation has two unknowns, and the highest degree of all terms is 1, this equation is called a linear equation in two unknowns.
- After marking all solutions of a linear equation in two unknowns on a rectangular coordinate plane, a straight line can be obtained. This straight line is called the graph of a linear equation in two unknowns.
- The coordinates of all the points on the graph of a linear equation in two unknowns are solutions of the equation. Conversely, all ordered pairs which satisfy the equation are coordinates of points lying on the graph of the equation.

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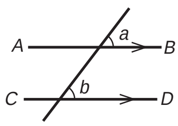
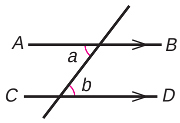
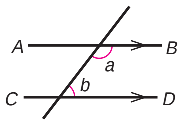
Angles in Rectilinear Figures
(S1B Chapter 13)



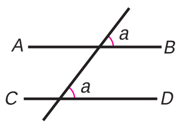
$a = b$
[corr. ∠s, AB // CD]



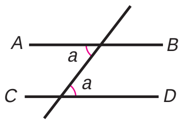
$a = b$
[alt. ∠s, AB // CD]



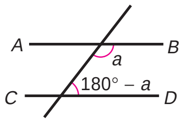
$a + b = 180°$
[int. ∠s, AB // CD]



AB // CD
[corr. ∠s eq.]



AB // CD
[alt. ∠s eq.]



AB // CD
[int. ∠s supp.]

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Ratio and Rate
(S1B Chapter 12)

- Ratio is the quotient of two quantities of the same kind, it has no units. The ratio of x to y is expressed as $x : y$ or $\frac{x}{y}$, where $x \neq 0$ and $y \neq 0$.
- Rate is the comparison of two quantities by division.

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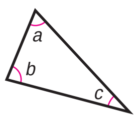
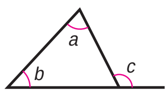
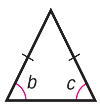
Angles in Rectilinear Figures
(S1B Chapter 13)



$a + b + c = 180°$
[∠ sum of Δ]



$c = a + b$
[ext. ∠ of Δ]



$b = c$
[base ∠s, isos. Δ]

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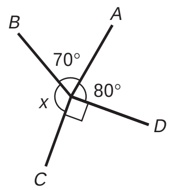
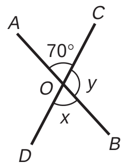
Example

Find the unknowns in the following figures.

(a)



(b) AOB and COD are straight lines



Solution:

(a) $x+70°+80°+90°=360°$ (∠s at a pt.)

$x+240°=360°$

$x=\underline{\underline{120°}}$

(b) $x=\underline{\underline{70°}}$ (vert. opp. ∠s)

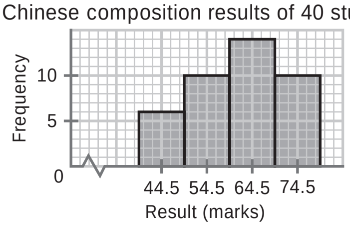
$y+70°=180°$ (adj. ∠s on st. line)

$y=\underline{\underline{110°}}$

Example

The following histogram shows the Chinese composition results of 40 students. It is known that the first class interval is 40 marks - 49 marks. How many students with their results reach 59.5 marks?

Chinese composition results of 40 students



Solution:

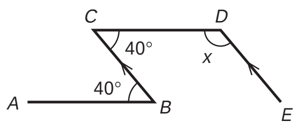
14 + 10 = $\underline{\underline{24}}$ students with their results reach 59.5 marks.

Example

In the figure, $BC // ED$.

(a) Find x.

(b) Prove that $AB // CD$.



Solution:

(a) $x+40°=180°$ (int. ∠s, $BC // ED$)

$x=\underline{\underline{140°}}$

(b) ∵ $\angle DCB = \angle CBA$ (given)

∴ $AB // CD$ (alt. ∠s eq.)

Example

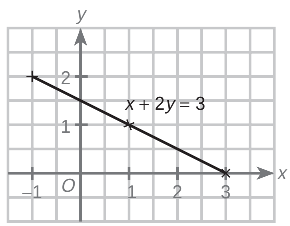
(a) Draw the graph of the equation $x + 2y = 3$ from $x = -1$ to $x = 3$ on a rectangular coordinate plane.

(b) Does $(-5, 1)$ lie on the graph of the equation $x + 2y = 3$?

Solution:

(a) $x + 2y = 3$

x	-1	1	3
y	2	1	0



(b) Substitute $x = -5$ and $y = 1$ into the equation,

L.H.S. = $x + 2y = -5 + 2(1) = -3$

R.H.S.= 3

∵ L.H.S. ≠ R.H.S.

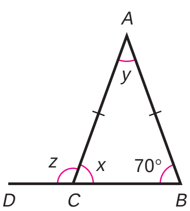
∴ $(-5, 1)$ does not satisfy the equation.

i.e. $\underline{\underline{\text{(−5, 1) does not lie on the graph of the equation } x + 2y = 3.}}$

Example

In the figure, BCD is a straight line, $AB = AC$.

Find x, y and z.



Solution:

∵ $AC = AB$ (given)

∴ $x=\underline{\underline{70°}}$ (base ∠s, isos. Δ)

$x+y+70°=180°$ (∠ sum of Δ)

$70°+y+70°=180°$

$y=\underline{\underline{40°}}$

$z=y+70°$ (ext. ∠ of Δ)

$=40°+70°$

$=\underline{\underline{110°}}$

Example

(a) In a museum, an adult ticket and a concessionary ticket are sold for \$30 and \$20 respectively. Find the ratio of the selling price of an adult ticket to that of a concessionary ticket.

(b) A box of oranges is sold for \$230. If there are 92 oranges in the box, express the selling price in \$/orange.

Solution:

(a) Ratio of the selling price of an adult ticket to that of a concessionary ticket

$=\frac{\$30}{\$20}$

$=\frac{3}{2}$

$=\underline{\underline{3:2}}$

(b) Selling price $=\frac{\$230}{92 \text{ oranges}}$

$=\underline{\underline{\$2.5/\text{orange}}}$